Advanced Calculus Foundations

Calculus · Answer Key · 25 Questions

1. Which theorem states that a continuous function on a closed interval [a, b] must attain its maximum and minimum values at least once?

A) Extreme Value Theorem
B) Intermediate Value Theorem
C) Mean Value Theorem
D) Bolzano-Weierstrass Theorem

2. What is the necessary and sufficient condition for a power series to converge uniformly within its radius of convergence?

A) The series must be alternating
B) The series must be absolute-convergent at the boundaries
C) Convergence must be uniform on every compact subset of the interior
D) The coefficients must be monotonic

3. In the context of the Riemann-Stieltjes integral, what condition must the integrator function alpha satisfy for the integral to exist for any continuous f?

A) Must be differentiable
B) Must be of bounded variation
C) Must be strictly increasing
D) Must be continuous

4. Which mathematical constant is defined as the limit of the sequence (1 + 1/n)^n as n approaches infinity?

A) Pi
B) Euler's number (e)
C) The Euler-Mascheroni constant
D) The Golden Ratio

5. What does the Cauchy-Hadamard theorem provide a formula for in the study of power series?

A) The interval of convergence
B) The radius of convergence
C) The number of roots
D) The point of inflection

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6. Which property is characteristic of a function that is Riemann integrable on [a, b]?

A) It must be differentiable everywhere
B) It must be continuous everywhere
C) The set of its discontinuities must have Lebesgue measure zero
D) It must be monotonic

7. What is the value of the derivative of the inverse function f^-1(y) at y = f(x) given f'(x) is non-zero?

A) f'(x)
B) 1 / f'(x)
C) f(x) * f'(x)
D) e^f'(x)

8. Which theorem establishes the relationship between a line integral around a simple closed curve and a double integral over the plane region it encloses?

A) Stokes' Theorem
B) Divergence Theorem
C) Green's Theorem
D) Taylor's Theorem

9. According to the Taylor series expansion, what is the value of the remainder term R_n(x) using the Lagrange form?

A) f^(n+1)(c) * (x-a)^(n+1) / (n+1)!
B) f^(n)(c) * (x-a)^n / n!
C) f^(n+1)(a) * (x-a)^(n+1) / (n+1)!
D) f(n)(c) * x^n / n!

10. What is the definition of a function being 'Lipschitz continuous' on an interval I?

A) It is uniformly continuous
B) It has a bounded derivative
C) There exists a constant K such that |f(x)-f(y)| <= K|x-y|
D) It is infinitely differentiable

11. Which test is used to determine the convergence of an infinite series by comparing the limit of the ratio of successive terms to 1?

A) Comparison Test
B) Integral Test
C) D'Alembert's Ratio Test
D) Root Test

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12. What is the primary requirement for a function to be considered 'analytic' at a point?

A) It is defined on the whole real line
B) It can be represented by a power series in some neighborhood of the point
C) It is Lebesgue integrable
D) It is bounded

13. The Weierstrass Approximation Theorem states that any continuous function on a closed interval can be uniformly approximated by which type of function?

A) Trigonometric series
B) Polynomials
C) Rational functions
D) Logarithmic functions

14. What is the result of the improper integral from 0 to infinity of e^(-x^2) dx?

A) sqrt(pi)
B) sqrt(pi)/2
C) pi/2
D) 1

15. Which criterion is used to prove that a sequence of real numbers converges if it is Cauchy?

A) Completeness of the real numbers
B) Bolzano-Weierstrass Theorem
C) Monotone Convergence Theorem
D) Heine-Borel Theorem

16. In vector calculus, what is the divergence of the curl of any twice continuously differentiable vector field?

A) The gradient of the field
B) The curl of the gradient
C) Zero
D) The Laplacian

17. What determines the existence of a unique solution to a first-order ordinary differential equation according to the Picard-Lindelöf theorem?

A) Lipschitz continuity of the function
B) The function must be linear
C) The initial value must be zero
D) The derivative must be constant

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18. What is the value of the Gamma function ?(n) when n is a positive integer?

A) n^n

B) (n-1)!

C) n!

D) e^n

19. The Fundamental Theorem of Calculus Part 1 establishes that if f is continuous, then the function defined by the integral of f is what?

A) An antiderivative of f

B) A constant function

C) The derivative of f

D) A periodic function

20. What does the Mean Value Theorem require for a function f on [a, b]?

A) f must be differentiable on [a, b]

B) f must be continuous on [a, b] and differentiable on (a, b)

C) f must be twice differentiable

D) f must be analytic

21. What is the value of the derivative of ln(x) at x = 1?

A) 0

B) 1

C) e

D) 1/e

22. Which series is the standard example of a divergent series despite its terms approaching zero?

A) Geometric series with r < 1

B) Harmonic series

C) Alternating series

D) Power series

23. The Bolzano-Weierstrass Theorem states that every bounded sequence in R has what?

A) A finite sum

B) A convergent subsequence

C) An upper bound

D) A derivative

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24. Which condition must be met for a function of two variables to have equal mixed partial derivatives?

A) Clairaut's Theorem (f must be C^2)

B) The function must be linear

C) The function must be periodic

D) The domain must be compact

25. What is the radius of convergence of the Taylor series for e^x centered at 0?

A) 0

B) 1

C) e

D) Infinity