

Algebraic Principles in Astrophysics

Mathematics · Answer Key · 15 Questions

1. If the orbital period T of a planet is proportional to the $3/2$ power of its semi-major axis a ($T = ka^{3/2}$), and a dwarf planet has a semi-major axis 4 times that of Earth, what is its orbital period in Earth years?

- A) 4 years
- B) 8 years**
- C) 16 years
- D) 32 years

2. The apparent magnitude m and absolute magnitude M of a star are related by $m - M = 5\log_{10}(d) - 5$, where d is the distance in parsecs. If a star has $m=5$ and $M=0$, what is its distance d in parsecs?

- A) 10 pc**
- B) 50 pc
- C) 100 pc
- D) 500 pc

3. The Schwarzschild radius R_s of a black hole is calculated by $R_s = 2GM/c^2$. If mass M is doubled, by what factor does the radius R_s increase?

- A) 0.5
- B) 1
- C) 2**
- D) 4

4. The Stefan-Boltzmann law states luminosity $L = A\sigma T^4$. If a star's radius is doubled while its surface temperature T remains constant, by what factor does its luminosity L increase?

- A) 2
- B) 4**
- C) 8
- D) 16

5. Escape velocity v is proportional to the square root of the mass M of a planet ($v = k\sqrt{M}$). If planet A has 9 times the mass of planet B, how many times greater is the escape velocity of A compared to B?

- A) 3**
- B) 6
- C) 9
- D) 81

6. Using the equation $d = vt$, if a radio signal travels at 300,000 km/s and takes 500 seconds to reach Earth from the Sun, what is the distance in kilometers expressed in scientific notation?

- A) 1.5×10^7 km
- B) 1.5×10^8 km**
- C) 3.0×10^8 km
- D) 1.5×10^9 km

7. The orbital velocity v of a satellite is given by $v = \sqrt{GM/r}$. If the distance r from the center of the planet is increased by a factor of 4, what happens to the orbital velocity v ?

- A) It decreases by half**
- B) It decreases by a quarter
- C) It increases by half
- D) It remains the same

8. The gravitational force F between two bodies is $F = G(m_1 m_2)/r^2$. If the distance r between two planets is tripled, the gravitational force F changes by what factor?

- A) $1/3$
- B) $1/6$
- C) $1/9$**
- D) 9

9. If a galaxy's redshift z is related to its recessional velocity v by $v = cz$, and c is 300,000 km/s, what is the velocity of a galaxy with a redshift of 0.05?

- A) 1,500 km/s
- B) 15,000 km/s**
- C) 30,000 km/s
- D) 150,000 km/s

10. The total energy E of a photon is $E = hc/\lambda$. If the wavelength λ is halved, what happens to the energy E of the photon?

- A) It is halved
- B) It remains the same
- C) It doubles**
- D) It quadruples

11. The density ρ of a planet is M/V . Given $V = \frac{4}{3}\pi r^3$, if the radius r of a planet is doubled while keeping mass M constant, the density ρ changes by what factor?

A) $1/8$

B) $1/4$

C) $1/2$

D) 2

12. Using the formula for the circumference of an orbit $C = 2\pi r$, if the average distance of Mars from the Sun is 228 million km, approximately how far does it travel in one complete circular orbit?

A) 716 million km

B) 1.43 billion km

C) 2.86 billion km

D) 4.56 billion km

13. The time dilation factor γ is $1/\sqrt{1 - v^2/c^2}$. If $v = 0.6c$, what is the value of γ ?

A) 1.25

B) 1.33

C) 1.5

D) 2.0

14. If the brightness of a star follows the inverse-square law $B = L/(4\pi d^2)$, and distance d is increased by a factor of 10, how does the brightness B change?

A) It becomes $1/10$

B) It becomes $1/20$

C) It becomes $1/50$

D) It becomes $1/100$

15. Using the formula for kinetic energy $KE = \frac{1}{2}mv^2$, if an asteroid's velocity v is tripled, what happens to its kinetic energy?

A) It increases by 3 times

B) It increases by 6 times

C) It increases by 9 times

D) It increases by 27 times