

Cosmic Algebra

Algebra · Answer Key · 18 Questions

1. If the orbital period P of a planet in years is related to its semi-major axis a in AU by Kepler's Third Law ($P^2 = a^3$), calculate the semi-major axis of a planet with an orbital period of 64 years.

- A) 4 AU
- B) 8 AU**
- C) 16 AU
- D) 32 AU

2. The luminosity L of a star is proportional to the square of its radius R and the fourth power of its surface temperature T ($L = kR^2T^4$). If the temperature doubles and the radius remains constant, by what factor does the luminosity increase?

- A) 2
- B) 4
- C) 8
- D) 16**

3. The escape velocity v of a planet is given by $v = \sqrt{2GM/r}$. If a planet's mass M is multiplied by 9 and its radius r is multiplied by 4, by what factor does the escape velocity change?

- A) 1.5**
- B) 2.25
- C) 3
- D) 4.5

4. Using the inverse-square law for light ($I = P / 4\pi d^2$), if the distance d to a star is tripled, by what factor does the observed intensity I decrease?

- A) $1/3$
- B) $1/6$
- C) $1/9$**
- D) $1/27$

5. The Schwarzschild radius R_s of a black hole is calculated as $2GM/c^2$. If you double the mass M of a black hole, how does the Schwarzschild radius change?

- A) It remains the same
- B) It doubles**
- C) It quadruples
- D) It halves

6. A satellite orbits Earth at a velocity v proportional to $1/\sqrt{r}$. If the orbital radius r is increased by a factor of 4, what is the new velocity relative to the original?

A) $\frac{1}{2} v$

B) $2 v$

C) $\frac{1}{4} v$

D) $4 v$

7. The gravitational force F between two objects is $F = G(m_1 m_2)/r^2$. If the mass of both objects is doubled and the distance between them is halved, by what factor does the force increase?

A) 4

B) 8

C) 16

D) 32

8. The apparent magnitude m of a star is related to its distance d in parsecs by $m = M + 5 \log_{10}(d) - 5$. If the distance d is 100 parsecs, what is the value of $5 \log_{10}(d)$?

A) 5

B) 10

C) 15

D) 20

9. The period of a simple pendulum on a planet is $T = 2\pi \sqrt{L/g}$. If gravity g on the planet is 4 times that of Earth, what is the ratio of the period on that planet to the period on Earth?

A) 1:2

B) 1:4

C) 2:1

D) 4:1

10. Hubble's Law states $v = H_0 * d$. If a galaxy is 500 Megaparsecs away and H_0 is 70 km/s/Mpc, what is its recessional velocity v ?

A) 350 km/s

B) 3,500 km/s

C) 35,000 km/s

D) 350,000 km/s

11. The surface area of a spherical planet is $A = 4\pi r^2$. If you increase the radius of the planet by a factor of 10, how does the surface area change?

- A) 10 times
- B) 20 times
- C) 100 times**
- D) 400 times

12. The volume of a gas cloud is $V = \frac{4}{3}\pi r^3$. If the radius of the cloud triples, what is the ratio of the new volume to the original volume?

- A) 3:1
- B) 9:1
- C) 27:1**
- D) 81:1

13. In the equation $E = mc^2$, if mass m is converted to energy entirely and the speed of light c is approximated as 3×10^8 m/s, what is the factor by which mass is multiplied to find energy?

- A) 9×10^{16}**
- B) 3×10^8
- C) 6×10^{16}
- D) 9×10^8

14. The flux of radiation from a star is $F = \sigma T^4$. If the temperature T of the star increases from 5,000K to 10,000K, the flux increases by what factor?

- A) 2
- B) 4
- C) 8
- D) 16**

15. The density ρ of a planet is $M / (\frac{4}{3}\pi r^3)$. If you double the radius while keeping the mass constant, what is the new density relative to the original?

- A) $1/2$
- B) $1/4$
- C) $1/8$**
- D) $1/16$

16. Centripetal force is $F = mv^2/r$. If a moon's velocity v is doubled and its orbit radius r remains constant, by what factor does the centripetal force increase?

- A) 2
- B) 4**
- C) 8
- D) 16

17. The Stefan-Boltzmann law relates total power P to area A and temperature T as $P = A\sigma T^4$. If a star's radius is halved and its temperature is doubled, what happens to the power P ?

- A) P is halved
- B) P is doubled
- C) P is quadrupled**
- D) P remains the same

18. The orbital energy of a satellite is $E = -GMm/2r$. If the distance r is doubled, what happens to the total orbital energy E ?

- A) It becomes half as negative**
- B) It becomes twice as negative
- C) It becomes four times as negative
- D) It remains the same