

Advanced Geometric Theories and Applications

Geometry · Answer Key · 10 Questions

1. Which of the following describes a fundamental property of the Poincaré conjecture, now proven as the Poincaré theorem?

- A) Every simply connected, closed 3-manifold is homeomorphic to the 3-sphere.**
- B) Any two points in a compact hyperbolic manifold can be joined by a unique geodesic.
- C) The sum of the interior angles of a spherical triangle is always greater than π radians.
- D) The Gaussian curvature of a minimal surface embedded in Euclidean 3-space is zero everywhere.

2. What is the mathematical significance of Hilbert's third problem concerning Dehn invariants?

- A) It establishes that two polyhedra of equal volume are equidecomposable if and only if they have the same Dehn invariant.**
- B) It proves the existence of a regular polyhedron for every integer number of faces.
- C) It demonstrates that a sphere cannot be decomposed into a finite number of pieces and reassembled into a cube of the same volume.
- D) It relates the area of a triangle to the squares of its sides using a constant invariant.

3. According to the Gauss-Bonnet theorem, what is the relationship between the integral of Gaussian curvature over a compact, orientable 2-manifold with boundary and its Euler characteristic?

- A) The integral of the Gaussian curvature equals 2π minus the Euler characteristic multiplied by 2π .**
- B) The integral of the Gaussian curvature is directly proportional to the Euler characteristic.
- C) The integral of the Gaussian curvature is always zero for any such manifold.
- D) The integral of the Gaussian curvature is equal to the Euler characteristic plus 2π .

4. In the context of differential geometry, what does the Ricci curvature tensor measure?

- A) The difference between the Riemann curvature tensor and the sectional curvature.
- B) The average sectional curvature of a Riemannian manifold at a point.**
- C) The deviation of a manifold from being flat in all directions.
- D) The scalar curvature divided by the dimension of the manifold.

5. What is the essence of Fenchel's theorem concerning the total curvature of a closed convex curve in the plane?

A) The total curvature of any simple closed convex curve in the plane is exactly 2π .

- B) The total curvature is inversely proportional to the radius of curvature.
- C) The total curvature is equal to the perimeter of the curve.
- D) The total curvature can be any positive real number.

6. Which theorem proves that any simple polygon can be triangulated by adding non-intersecting diagonals?

A) The Polygon Triangulation Theorem.

- B) The Shoelace Formula.
- C) The Two Ears Theorem.
- D) The Helly's Theorem.

7. What geometric principle underlies the construction of a hyperbolic paraboloid?

A) It can be generated by moving a straight line along another straight line while maintaining a constant angle.

- B) It is formed by the intersection of a sphere and a plane.
- C) It is defined by the locus of points equidistant from a point and a plane.
- D) It is the result of rotating an ellipse around an axis.

8. What is the fundamental statement of the Isoperimetric Inequality for a simple closed curve in the Euclidean plane?

A) Among all curves of a given perimeter, the circle encloses the maximum area.

- B) The area enclosed by a curve is always greater than its perimeter.
- C) For a given area, the square has the minimum perimeter.
- D) The ratio of perimeter squared to area is minimized by a square.

9. According to the Baire Category Theorem, in a complete metric space, the intersection of a countable collection of dense open sets is:

A) Dense.

- B) Empty.
- C) A single point.
- D) Compact.

10. What is the definition of the Euler characteristic for a convex polytope in three dimensions?

A) The number of vertices minus the number of edges plus the number of faces.

- B) The sum of the number of vertices and faces.
- C) Twice the number of edges minus the number of vertices.
- D) The number of faces minus the number of edges.