

Advanced Number Theory Concepts

Number Theory · Practice Test · 19 Questions

1. Which theorem states that for any integer $n > 1$, there exists a prime p such that $n < p < 2n$?

- A) Dirichlet's theorem on arithmetic progressions
- B) Bertrand's Postulate
- C) The Prime Number Theorem
- D) Lagrange's Four-Square Theorem

2. The Riemann Hypothesis, if proven true, would have profound implications for the distribution of which mathematical objects?

- A) Perfect numbers
- B) Mersenne primes
- C) Composite numbers
- D) Zeros of the Riemann zeta function

3. What is the smallest positive integer that is a perfect cube and also the sum of two positive integer cubes?

- A) 1729
- B) 4104
- C) 13832
- D) 20683

4. According to the Frobenius Coin Problem, for a set of relatively prime positive integers $\{a_1, a_2, \dots, a_n\}$, what is the largest integer that cannot be expressed as a non-negative integer linear combination of these integers?

- A) The Frobenius number
- B) The least common multiple
- C) The greatest common divisor
- D) The sum of the integers

5. What property does a number n possess if it is a primitive root modulo p , where p is a prime number?

- A) It divides $p-1$
- B) Its order modulo p is $p-1$
- C) It is congruent to 1 modulo p
- D) It is congruent to -1 modulo p

6. Which theorem establishes that every positive integer can be written as the sum of at most four integer squares?

- A) Fermat's theorem on sums of two squares
- B) Lagrange's four-square theorem
- C) Gauss's lemma on quadratic residues
- D) The fundamental theorem of arithmetic

7. The classification of finite simple groups, a monumental achievement in mathematics, was completed in the 20th century and involves groups related to which number-theoretic concept?

- A) Prime numbers
- B) Diophantine equations
- C) Modular forms
- D) Cyclotomic fields

8. What is the significance of the number 1 for the set of prime numbers?

- A) It is the smallest prime number.
- B) It is a prime number by definition.
- C) It is not considered a prime number because it has only one divisor.
- D) It is the only even prime number.

9. The existence of infinitely many prime numbers was first proven by which ancient Greek mathematician?

- A) Pythagoras
- B) Archimedes
- C) Euclid
- D) Diophantus

10. What is the definition of a Carmichael number?

- A) A composite number n such that a^{n-1} is congruent to 1 (mod n) for all integers a with $\gcd(a, n) = 1$.
- B) A prime number p such that $2^p - 1$ is also prime.
- C) A number that is the sum of its proper divisors.
- D) A number whose decimal representation terminates.

11. The class number problem in algebraic number theory is concerned with the number of what in the ideal class group of a number field?

- A) Units
- B) Prime ideals
- C) Ideal classes
- D) Roots of unity

12. Which result from analytic number theory provides an asymptotic formula for the number of primes less than or equal to x ?

- A) Dirichlet's theorem on arithmetic progressions
- B) The Prime Number Theorem
- C) Minkowski's theorem
- D) The density hypothesis

13. What is the main subject of Fermat's Last Theorem, proven by Andrew Wiles?

- A) The impossibility of integer solutions to $x^n + y^n = z^n$ for $n > 2$.
- B) The infinitude of prime numbers.
- C) The existence of a smallest composite number.
- D) The sum of divisors function.

14. The concept of quadratic reciprocity, elegantly expressed by Gauss, relates the solvability of which type of congruence?

- A) Linear congruences
- B) Quadratic congruences
- C) Cubic congruences
- D) Higher-order congruences

15. What is the smallest perfect number?

- A) 28
- B) 6
- C) 496
- D) 8128

16. The study of Diophantine equations primarily concerns integer solutions to which type of equations?

- A) Differential equations
- B) Polynomial equations
- C) Trigonometric equations
- D) Logarithmic equations

17. What is the name of the conjecture that states every even integer greater than 2 is the sum of two primes?

- A) Goldbach's Conjecture
- B) Twin Prime Conjecture
- C) Mersenne Conjecture
- D) Catalan's Conjecture

18. The structure of the multiplicative group of integers modulo n , denoted by $(\mathbb{Z}/n\mathbb{Z})^*$, is fundamentally determined by what property of n ?

- A) Its sum of divisors
- B) Its prime factorization
- C) Its number of digits
- D) Its magnitude

19. Which theorem states that if p is a prime number, then for any integer a , a^p is congruent to a modulo p ?

- A) Fermat's Little Theorem
- B) Euler's Totient Theorem
- C) Wilson's Theorem
- D) Chinese Remainder Theorem