Advanced Number Theory Concepts

Number Theory · Answer Key · 19 Questions

1. Which theorem states that for any integer n > 1, there exists a prime p such that n < p < 2n?

A) Dirichlet's theorem on arithmetic progressions
B) Bertrand's Postulate
C) The Prime Number Theorem
D) Lagrange's Four-Square Theorem

2. The Riemann Hypothesis, if proven true, would have profound implications for the distribution of which mathematical objects?

A) Perfect numbers
B) Mersenne primes
C) Composite numbers
D) Zeros of the Riemann zeta function

3. What is the smallest positive integer that is a perfect cube and also the sum of two positive integer cubes?

A) 1729
B) 4104
C) 13832
D) 20683

4. According to the Frobenius Coin Problem, for a set of relatively prime positive integers {a1, a2, ..., an}, what is the largest integer that cannot be expressed as a non-negative integer linear combination of these integers?

A) The Frobenius number
B) The least common multiple
C) The greatest common divisor
D) The sum of the integers

5. What property does a number n possess if it is a primitive root modulo p, where p is a prime number?

A) It divides p-1
B) Its order modulo p is p-1
C) It is congruent to 1 modulo p
D) It is congruent to -1 modulo p

6. Which theorem establishes that every positive integer can be written as the sum of at most four integer squares?

A) Fermat's theorem on sums of two squares
B) Lagrange's four-square theorem
C) Gauss's lemma on quadratic residues
D) The fundamental theorem of arithmetic

7. The classification of finite simple groups, a monumental achievement in mathematics, was completed in the 20th century and involves groups related to which number-theoretic concept?

A) Prime numbers
B) Diophantine equations
C) Modular forms
D) Cyclotomic fields

8. What is the significance of the number 1 for the set of prime numbers?

A) It is the smallest prime number.
B) It is a prime number by definition.
C) It is not considered a prime number because it has only one divisor.
D) It is the only even prime number.

9. The existence of infinitely many prime numbers was first proven by which ancient Greek mathematician?

A) Pythagoras
B) Archimedes
C) Euclid
D) Diophantus

10. What is the definition of a Carmichael number?

A) A composite number n such that a^(n-1) is congruent to 1 (mod n) for all integers a with gcd(a, n) = 1.
B) A prime number p such that 2^p - 1 is also prime.
C) A number that is the sum of its proper divisors.
D) A number whose decimal representation terminates.

11. The class number problem in algebraic number theory is concerned with the number of what in the ideal class group of a number field?

A) Units
B) Prime ideals
C) Ideal classes
D) Roots of unity

12. Which result from analytic number theory provides an asymptotic formula for the number of primes less than or equal to x?

A) Dirichlet's theorem on arithmetic progressions

B) The Prime Number Theorem

C) Minkowski's theorem

D) The density hypothesis

13. What is the main subject of Fermat's Last Theorem, proven by Andrew Wiles?

A) The impossibility of integer solutions to x^n + y^n = z^n for n > 2.

B) The infinitude of prime numbers.

C) The existence of a smallest composite number.

D) The sum of divisors function.

14. The concept of quadratic reciprocity, elegantly expressed by Gauss, relates the solvability of which type of congruence?

A) Linear congruences

B) Quadratic congruences

C) Cubic congruences

D) Higher-order congruences

15. What is the smallest perfect number?

A) 28

B) 6

C) 496

D) 8128

16. The study of Diophantine equations primarily concerns integer solutions to which type of equations?

A) Differential equations

B) Polynomial equations

C) Trigonometric equations

D) Logarithmic equations

17. What is the name of the conjecture that states every even integer greater than 2 is the sum of two primes?

A) Goldbach's Conjecture

B) Twin Prime Conjecture

C) Mersenne Conjecture

D) Catalan's Conjecture

18. The structure of the multiplicative group of integers modulo n, denoted by (Z/nZ)*, is fundamentally determined by what property of n?

A) Its sum of divisors

B) Its prime factorization

C) Its number of digits

D) Its magnitude

19. Which theorem states that if p is a prime number, then for any integer a, a^p is congruent to a modulo p?

A) Fermat's Little Theorem

B) Euler's Totient Theorem

C) Wilson's Theorem

D) Chinese Remainder Theorem