

Advanced Algebraic Theory

Algebra · Practice Test · 12 Questions

1. Which theorem states that every non-constant polynomial with coefficients in the field of complex numbers has at least one root in the field of complex numbers?

- A) Fundamental Theorem of Algebra
- B) Lagrange's Theorem
- C) Sylow's Theorem
- D) Cauchy's Theorem

2. In group theory, what is the order of the alternating group A_n (the group of even permutations of n elements) for $n \geq 2$?

- A) n
- B) $n!$
- C) $n!/2$
- D) $(n-1)!$

3. Which field is defined as the smallest field extension of the rational numbers that contains all the roots of a given polynomial?

- A) Cyclotomic field
- B) Splitting field
- C) Galois field
- D) Algebraic closure

4. What is the characteristic of a finite field with p^n elements, where p is a prime number?

- A) n
- B) p
- C) p^n
- D) $p+n$

5. According to the Abel-Ruffini theorem, why is there no general algebraic solution for polynomial equations of degree five or higher?

- A) Lack of sufficient field extensions
- B) The Galois group is not solvable
- C) The discriminant is always zero
- D) The coefficients are transcendental

6. What is the structure of the group of units of the ring of integers modulo n , denoted as $(\mathbb{Z}/n\mathbb{Z})^\times$, when n is a prime p ?

- A) Cyclic group of order p
- B) Cyclic group of order $p-1$
- C) Symmetric group S_p
- D) Abelian group of order p

7. In the context of ring theory, what is a ring in which every ideal is a principal ideal called?

- A) Euclidean domain
- B) Principal Ideal Domain
- C) Unique Factorization Domain
- D) Field

8. Which condition must a ring satisfy to be classified as a division ring?

- A) It must be commutative
- B) It must contain zero divisors
- C) Every non-zero element must have a multiplicative inverse
- D) It must be a finite field

9. What is the dimension of the vector space of 3×3 matrices over a field F ?

- A) 3
- B) 6
- C) 8
- D) 9

10. In Galois theory, the Galois group of a field extension is defined as the group of all automorphisms of the extension field that fix which subfield?

- A) The base field
- B) The algebraic closure
- C) The field of complex numbers
- D) The field of integers

11. Which of the following is a necessary condition for a group to be considered a simple group?

- A) It must be abelian
- B) It has no non-trivial normal subgroups
- C) It must be of prime order
- D) It must be a cyclic group

12. What is the minimum polynomial of the complex number i (the square root of -1) over the field of rational numbers?

- A) $x - i$
- B) $x + 1$
- C) $x^2 - 1$
- D) $x^2 + 1$