

Advanced Algebraic Theory

Algebra · Answer Key · 12 Questions

1. Which theorem states that every non-constant polynomial with coefficients in the field of complex numbers has at least one root in the field of complex numbers?

- A) Fundamental Theorem of Algebra**
- B) Lagrange's Theorem
- C) Sylow's Theorem
- D) Cauchy's Theorem

2. In group theory, what is the order of the alternating group A_n (the group of even permutations of n elements) for $n \geq 2$?

- A) n
- B) $n!$
- C) $n!/2$**
- D) $(n-1)!$

3. Which field is defined as the smallest field extension of the rational numbers that contains all the roots of a given polynomial?

- A) Cyclotomic field
- B) Splitting field**
- C) Galois field
- D) Algebraic closure

4. What is the characteristic of a finite field with p^n elements, where p is a prime number?

- A) n
- B) p**
- C) p^n
- D) $p+n$

5. According to the Abel-Ruffini theorem, why is there no general algebraic solution for polynomial equations of degree five or higher?

- A) Lack of sufficient field extensions
- B) The Galois group is not solvable**
- C) The discriminant is always zero
- D) The coefficients are transcendental

6. What is the structure of the group of units of the ring of integers modulo n , denoted as $(\mathbb{Z}/n\mathbb{Z})^\times$, when n is a prime p ?

A) Cyclic group of order p

B) Cyclic group of order $p-1$

C) Symmetric group S_p

D) Abelian group of order p

7. In the context of ring theory, what is a ring in which every ideal is a principal ideal called?

A) Euclidean domain

B) Principal Ideal Domain

C) Unique Factorization Domain

D) Field

8. Which condition must a ring satisfy to be classified as a division ring?

A) It must be commutative

B) It must contain zero divisors

C) Every non-zero element must have a multiplicative inverse

D) It must be a finite field

9. What is the dimension of the vector space of 3×3 matrices over a field F ?

A) 3

B) 6

C) 8

D) 9

10. In Galois theory, the Galois group of a field extension is defined as the group of all automorphisms of the extension field that fix which subfield?

A) The base field

B) The algebraic closure

C) The field of complex numbers

D) The field of integers

11. Which of the following is a necessary condition for a group to be considered a simple group?

A) It must be abelian

B) It has no non-trivial normal subgroups

C) It must be of prime order

D) It must be a cyclic group

12. What is the minimum polynomial of the complex number i (the square root of -1) over the field of rational numbers?

A) $x - i$

B) $x + 1$

C) $x^2 - 1$

D) $x^2 + 1$