

Astrophysical Algebra

Algebra · Answer Key · 24 Questions

1. If the orbital period T of a planet in years is given by $T = a^{(3/2)}$ where 'a' is the semi-major axis in AU, what is the semi-major axis of a planet with an orbital period of 64 years?

- A) 4 AU
- B) 8 AU
- C) 16 AU**
- D) 32 AU

2. The luminosity L of a star is proportional to the square of its radius R and the fourth power of its temperature T ($L \propto R^2 T^4$). If a star's radius is doubled and temperature halved, by what factor does its luminosity change?

- A) 0.5
- B) 1**
- C) 2
- D) 4

3. The escape velocity v from a planet is given by $v = \sqrt{2GM/r}$. If mass M is increased by 8 and radius r by 2, by what factor does the escape velocity increase?

- A) 1
- B) 2**
- C) 4
- D) 8

4. Using the equation $d = vt$, where d is distance and v is velocity, calculate the distance in km to a star if light (300,000 km/s) takes 5 years to reach Earth (Assume 1 year = 31,536,000 seconds).

- A) 4.73×10^{13} km**
- B) 9.46×10^{13} km
- C) 1.42×10^{14} km
- D) 2.84×10^{14} km

5. The magnitude system for star brightness uses the formula $m_1 - m_2 = -2.5 \log_{10}(F_1/F_2)$. If the flux ratio F_1/F_2 is 100, what is the difference in magnitude?

- A) 1
- B) 2.5
- C) 5**
- D) 10

6. A satellite orbits at a radius r . The centripetal acceleration is $a = v^2/r$. If velocity v is tripled and radius r is doubled, what is the new acceleration factor?

- A) 1.5
- B) 4.5**
- C) 6
- D) 9

7. The gravitational force F between two bodies is $F = G(m_1*m_2)/r^2$. If the distance r between Earth and the Moon were halved, how would the gravitational force change?

- A) Halved
- B) Doubled
- C) Quartered
- D) Quadrupled**

8. The Drake Equation estimates $N = R * f_p * n_e * f_l * f_i * f_c * L$. If all variables are constants except L (the lifetime of a civilization), and L increases by a factor of 10, how does N change?

- A) Remains same
- B) Increases by 10**
- C) Increases by 100
- D) Decreases by 10

9. Hubble's Law states $v = H_0 * d$. If H_0 is 70 km/s/Mpc, at what distance in Mpc is a galaxy moving away at 7,000 km/s?

- A) 10 Mpc
- B) 50 Mpc
- C) 100 Mpc**
- D) 700 Mpc

10. The surface area of a spherical planet is $4\pi r^2$. If the radius of Mars is approximately 3,390 km, what is the expression for its surface area?

- A) 11,492,100? km²
- B) 45,968,400? km²**
- C) 12,852,100? km²
- D) 38,556,300? km²

11. The redshift z is defined by $1+z = \lambda_{obs} / \lambda_{emit}$. If a galaxy has a spectral line observed at 600nm that was emitted at 400nm, what is the redshift z ?

- A) 0.25
- B) 0.5**
- C) 0.75
- D) 1.5

12. For an object in circular orbit, orbital velocity $v = \sqrt{GM/r}$. If a planet moves to a new orbit where the radius is 4 times larger, what is the factor of change in orbital velocity?

- A) 0.25
- B) 0.5**
- C) 2
- D) 4

13. The mass-luminosity relation for main sequence stars is $L \propto M^{3.5}$. If a star is 4 times as massive as the Sun, how many times more luminous is it?

- A) 14
- B) 32
- C) 64
- D) 128**

14. A rocket's mass ratio is $R = m_i/m_f$. Using the Tsiolkovsky rocket equation $\Delta v = v_e \ln(R)$, if $\Delta v = 2v_e$, what is the mass ratio R ?

- A) e
- B) e^2**
- C) $2e$
- D) $\ln(2)$

15. The Stefan-Boltzmann law states $P = A\sigma T^4$. If the temperature of a blackbody increases from 2,000K to 4,000K, by what factor does the power output increase?

- A) 2
- B) 4
- C) 8
- D) 16**

16. The orbital angular momentum $L = mvr$. If a satellite's mass is doubled, its velocity is halved, and its distance from the planet is tripled, what is the new angular momentum?

- A) 1.5 times the original**
- B) 2 times the original
- C) 3 times the original
- D) 4.5 times the original

17. Kepler's Second Law implies that for a planet in an elliptical orbit, the product of velocity and distance at perihelion equals the product at aphelion ($v_1 r_1 = v_2 r_2$). If $r_1=1\text{AU}$ and $r_2=3\text{AU}$, what is the ratio v_1/v_2 ?

- A) 1:3
- B) 3:1**
- C) 1:9
- D) 9:1

18. The density of a spherical planet is $\rho = M / (4/3 \pi r^3)$. If the radius of a planet is doubled while mass remains constant, what is the factor of change in density?

- A) 0.125**
- B) 0.25
- C) 0.5
- D) 4

19. If the Sun's mass is 1.989×10^{30} kg and Earth's mass is 5.97×10^{24} kg, what is the approximate ratio of the Sun's mass to Earth's mass?

- A) 3.33×10^5**
- B) 3.33×10^4
- C) 3.33×10^6
- D) 3.33×10^3

20. The Schwarzschild radius R_s of a black hole is $2GM/c^2$. If the mass M of a black hole is doubled, what is the factor of change in the Schwarzschild radius?

- A) 0.5
- B) 2**
- C) 4
- D) 8

21. The energy E of a photon is $E = hc/\lambda$. If the wavelength λ is cut in half, what happens to the energy E ?

- A) Divided by 2
- B) Multiplied by 2**
- C) Multiplied by 4
- D) Remains same

22. A light-year is the distance light travels in a year. Given $c = 3 \times 10^8$ m/s, approximately how many meters is one light-year?

A) 9.46×10^{15} m

B) 3×10^{12} m

C) 3.15×10^7 m

D) 9.46×10^{12} m

23. If the period of a pulsar is 0.033 seconds, what is its frequency in Hertz ($f = 1/T$)?

A) 3.03 Hz

B) 10.3 Hz

C) 30.3 Hz

D) 303 Hz

24. The flux of light from a star follows the inverse square law $F = L / (4\pi d^2)$. If the distance d is increased by 10, how does the flux F change?

A) Decreases by 10

B) Decreases by 100

C) Increases by 10

D) Increases by 100