

Advanced Statistics and Probability Concepts

Statistics & Probability · Answer Key · 18 Questions

1. Which theorem establishes that the distribution of the sample mean, when the sample size is sufficiently large, approaches a normal distribution, irrespective of the population's distribution?

- A) Bayes' Theorem
- B) The Law of Large Numbers
- C) The Central Limit Theorem**
- D) The Borel-Cantelli Lemma

2. In the context of hypothesis testing, what is the primary implication of a Type I error?

- A) Failing to reject a false null hypothesis.
- B) Rejecting a true null hypothesis.**
- C) Accepting a true alternative hypothesis.
- D) Failing to accept a false alternative hypothesis.

3. The Cauchy-Schwarz inequality for random variables X and Y states that $(E[XY])^2 \leq E[X^2]E[Y^2]$. What condition must hold for equality to occur?

- A) X and Y must be independent.
- B) X and Y must be identically distributed.
- C) One variable must be a linear function of the other ($Y = aX + b$).**
- D) The expected values of X and Y must be zero.

4. Which of the following probability distributions is characterized by its memoryless property, meaning the probability of future events is independent of past events?

- A) Binomial Distribution
- B) Poisson Distribution
- C) Geometric Distribution**
- D) Hypergeometric Distribution

5. What is the definition of a stationary process in time series analysis?

- A) A process where the mean and variance change over time.
- B) A process whose statistical properties, such as mean, variance, and autocorrelation, do not change over time.**
- C) A process that is strictly increasing.
- D) A process where all future values are predictable from past values.

6. The concept of 'sufficient statistic' in statistical inference means a statistic that:

- A) Is equal to the parameter being estimated.
- B) Contains all the information in the sample relevant to the parameter.**
- C) Is always normally distributed.
- D) Is independent of the parameter being estimated.

7. In Bayesian statistics, what is the posterior distribution?

- A) The prior belief about a parameter before observing data.
- B) The probability of observing the data given the parameter.
- C) The updated probability distribution of a parameter after considering observed data.**
- D) The probability of the parameter being true.

8. What is the fundamental property of a sigma-algebra (or Borel field) of events in probability theory?

- A) It contains only a finite number of events.
- B) It is closed under countable unions, countable intersections, and complements.**
- C) It includes every possible outcome.
- D) It only contains mutually exclusive events.

9. The principle of maximum likelihood estimation (MLE) aims to find the parameter values that:

- A) Minimize the variance of the estimator.
- B) Maximize the probability of observing the given sample data.**
- C) Minimize the bias of the estimator.
- D) Ensure the estimator is unbiased.

10. Kolmogorov's axioms of probability, foundational to modern probability theory, include that probability is non-negative, the probability of the sample space is 1, and:

- A) The probability of the union of two disjoint events is the product of their probabilities.
- B) The probability of the union of disjoint events is countably additive.**
- C) The probability of any event is between 0 and 1.
- D) The probability of an impossible event is 1.

11. What does the concept of 'conjugate prior' imply in Bayesian inference?

- A) The posterior distribution is identical to the prior distribution.
- B) The posterior distribution belongs to the same family of distributions as the prior.**
- C) The prior distribution is irrelevant to the posterior distribution.
- D) The posterior distribution is always uniform.

12. The Cramér-Rao lower bound provides a theoretical limit for the variance of any unbiased estimator of a parameter. An estimator achieving this bound is called:

- A) A consistent estimator.
- B) A maximum likelihood estimator.
- C) An efficient estimator.**
- D) A sufficient estimator.

13. In the context of multivariate statistics, what does the Jacobian matrix represent?

- A) The determinant of the covariance matrix.
- B) The rate of change of a vector-valued function.**
- C) The correlation between two variables.
- D) The probability density of a joint distribution.

14. What is the fundamental characteristic of a Poisson process?

- A) It models continuous events occurring at fixed intervals.
- B) It models the number of events occurring in a fixed interval of time or space, with events occurring independently and at a constant average rate.**
- C) It models the probability of success in a fixed number of trials.
- D) It models the time until the first success in a series of Bernoulli trials.

15. Which theorem states that for a sequence of independent and identically distributed random variables with finite variance, the sample mean converges in probability to the expected value?

- A) Central Limit Theorem
- B) Borel-Cantelli Lemma
- C) Law of Large Numbers**
- D) Slutsky's Theorem

16. What is the primary purpose of a p-value in hypothesis testing?

- A) To indicate the size of the effect.
- B) To represent the probability of obtaining test results at least as extreme as the results actually observed, assuming the null hypothesis is true.**
- C) To state the probability that the null hypothesis is true.
- D) To measure the power of the test.

17. The concept of ergodicity in stochastic processes implies that:

- A) The process's statistical properties are independent of time.
- B) Time averages are equal to ensemble averages.**
- C) The process has a constant variance.
- D) The process converges to a normal distribution.

18. What is the definition of a 'sufficient statistic' in statistical inference?

- A) A statistic whose distribution depends only on the parameter being estimated.
- B) A statistic that provides all the information about the parameter that the entire sample contains.**
- C) A statistic that is always an unbiased estimator.
- D) A statistic that is independent of the parameter being estimated.